

# Interpreting Negative Probabilities in the Context of Double-Slit Interferometry

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**Abstract.** Negative probabilities emerged at intermediate steps in various attempts to predict the distributions of quantum interference. There is no consensus on their meaning yet. It has been suggested (Khrennikov, 1998) that negative probabilities require the existence of unsuspected correlations between detection events. We evaluate this claim in light of several representative experiments. In our assessment, some of its implications are in good agreement with the data.

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## INTRODUCTION

It is a matter of common sense that an event can either happen or not. If its occurrence is certain, the rule is that its probability must be equal to 1. If it cannot happen, the probability must be equal to 0. In all other instances, the probability can have any value between 0 and 1. With this in mind, the idea that an event can have a negative probability of occurrence is non-sense. Or is it?

According to Richard Feynman, all negative numbers defy common sense in real-life situations. An apple trader may begin the day with 5 apples and end it with 3, after giving away 10 and receiving 8 apples. At no point in time was the trader in possession of a negative quantity of apples. Still, the analysis of the process as a whole is greatly simplified by showing that  $5 - 10 = -5$  and  $-5 + 8 = 3$ . Similarly, it must be acceptable to use negative probabilities as well, so long as they simplify thought and calculations in properly chosen situations. In his famous paper on negative probability [1], which he began with the apple trader example, Feynman says: “It is not my intention here to contend that the final probability of a verifiable physical event can be negative. On the other hand, conditional probabilities and probabilities of imagined intermediary states may be negative in a calculation of probabilities of physical events or states” (p. 238). Indeed, Feynman went on to conclude that “all the results of quantum statistics can be described in classical probability language, with states replaced by ‘conditions’ defined by a pair of states (or other variables), provided we accept negative values for these probabilities” (p. 248).

Negative probabilities are primarily useful for simplifying analysis, as confirmed by numerous other developments in modern physics. Yet, Richard Feynman went beyond mere calculations. He showed that negative probabilities have meta-theoretical implications as well. On the one hand, they can tell us something about the fitness of a theory. If a model predicts negative probabilities for real detectable states, it must be clearly wrong or incomplete. On the other hand, they can also tell us something about Nature. In some

contexts, there might be pairs of events, one of which has an abnormal probability value. This could mean that only one event is observable during a single act of measurement, in obvious agreement with Heisenberg's uncertainty principle. The difficulty, from our point of view, is that interpretation is not always straightforward. In many cases it is hard to say if a theory is incomplete, or if Nature simply defies common sense. For example, how does it help us to know that the distribution of detection events in a double-slit experiment can be predicted with models that involve negative probabilities?

Feynman's approach to negative probability was further refined in the context of double-slit interferometry by Scully, Walther and Schleich [2]. They showed that several questions, formulated by Feynman in abstract terms, can be illustrated physically with the help of the micromaser which-path detector. This allowed them to develop a very instructive formalism, and also to produce new insights into the EPR problem. Of interest to us is the general interpretation, implicit in this work. It shows that quantum properties are not reducible to classical phenomena (even though classical statistics can be used to describe it, as suggested by Feynman). The non-local interactions of quantum processes must be fundamental, providing a direct explanation for the fact that negative probabilities are required. In other words, the latter cannot be attributed to some sort of undiscovered physical interaction.

The above comments notwithstanding, Scully, Walther and Schleich did not attempt to provide any definitive interpretive conclusions. The problem of explaining the precise meaning of negative probabilities in the context of double-slit interference remained open. In this paper we shall focus on the attempt to find a plausible solution in terms of  $p$ -adic probability analysis, as formulated by Khrennikov [3]. This approach is very interesting because it shows that a classical interaction could still explain quantum interference. We shall provide a brief overview of the main claims and the initial difficulties associated with their verification. We shall also note the limitations of the original proposals and discuss a new way to verify this model. More importantly, we shall examine the results of several experiments, which appear to confirm the main prediction of this approach.

## SEARCHING FOR MEANING

Quantum statistics appeared to be easier to predict than to interpret. Khrennikov [3] noted that several earlier models had reproduced the distributions of the double-slit experiment successfully, but none of them was shown to have a clear interpretation. In contrast to these approaches, which were formulated as measure theories, he developed a theory in which probabilities were defined as relative frequencies. The outcome was a model with straightforward physical meaning, in part because of its innovative use of  $p$ -adic number analysis, which was not widely used before.

The main finding of this approach is that detection events in a double-slit experiment obey a logarithmic complexity rule. Independent random events obey linear complexity rules. Therefore, individual quanta cannot generate interference independently. They must display correlated behavior, in order to produce their well-known distributions. In other words, self-interference cannot be the reason for the detection pattern. Some sort of interaction, possibly classical, must occur. This prediction is particularly remarkable,

because it runs against both the Copenhagen interpretation and the most well-known pilot-wave models. Moreover, quantum interference was well demonstrated at low rates of emission, which was widely considered enough to rule out any direct interaction between single quanta. The concept of self-interference is pretty much taken for granted today.

Khrennikov showed that direct interaction between the quanta is not necessary for an interpretation of his results. The correlations required in this context could be produced by delayed (memory) effects within measurement systems. Such effects could happen either at the source of emission, or at the slits, or finally at the detectors. The proposition was easily testable. Yet, the problem was that the experiments did not seem to confirm it. Firstly, interference was demonstrated with pairs of independent deterministic sources of photons, which emit single pulses on demand [4]. This seems to be a convincing argument against correlation at the source. On the one hand, deterministic sources can have arbitrary moments of emission, which makes the idea of correlation between consecutive events implausible. On the other hand, the same experiment showed that fringe visibility depended primarily on the parameters of propagation inside the interferometer. Secondly, there are many types of interferometers which do not require slits. In some cases, even Young interference can happen with or without slits (as shown, for example, in ref. [5]). It is possible to argue that the slits are functionally similar to sources in many set-ups, which further confirms that memory effects at the openings are not essential, even if conceivably real. Finally, the hypothesis of memory effects at the detector is undermined by the wide use of modern (other than screen) detectors. For example, pico-streak cameras satisfy the requirement of having physical changes in the detector between detections, and they do not have diminished visibility. In fact, they can be used to observe important details, such as fringe drift, extending the class of observable interference phenomena (see for example, ref. [6]). More recent detection techniques, such as fiber-optic scanning, reinforce this conclusion even further, reducing the likely importance of memory effects during measurement. There are also experiments with neutron interferometers, quoted by Khrennikov *et al.* in a follow-up paper [7], which did not find the expected memory effects. Consequently, measurement artifacts (such as memory effects) cannot explain convincingly the peculiarities of non-classical distributions.

Given the above, it seems appropriate to follow the example of Feynman and ask: what if some of our assumptions are wrong? What if self-interference is not a real phenomenon, despite its popularity as an interpretive tool? After all, the simplest way to account for correlations among detection events is to assume that quanta interact with each other, even when they are not involved in direct collisions. Such a hypothesis might seem to go against the grain, but the relevant question is whether it is possible to formulate a plausible model for it. As shown elsewhere [8, 9], such a model can be developed in a way that is consistent with the relevant experimental evidence. If quanta are treated as sources of real waves, they can be shown to produce interference fringes without colliding directly. In the case of photons, the most important elements would be the length of pulses and the amplitude of created waves, which tends to diminish with distance from the source. This means that the reality of self-interference is testable by checking for its indicators in critical cases, where the predictions of different approaches do not converge. According to our analysis, the expected properties of self-interference did not materialize in relevant experiments. By implication, Khrennikov's conclusions are

strongly supported by empirical data. This means that quantum interference could really contain a hidden interaction, understandable in the language of classical mechanics. By switching the explanation of negative probabilities from the properties of individual quanta to the interactions among them, this approach has reopened the question of completeness of quantum mechanics. At least in the case of the double-slit experiment, an alternative interpretation became possible. We review a few of the most relevant experiments in the following chapter.

## INTERFEROMETRIC EVIDENCE

The hypothetical absence of self-interference has several experimental implications that have already been tested, as part of unrelated investigations [9]. Two of them are especially relevant for the present discussion. Firstly, correlations must vanish when their physical preconditions are not met. Below predictable energy levels, which translate into quantum density per volume of space-time, interference must become gradually undetectable. In the case of photons, whose action is proportional to their duration, these threshold rates must also depend on pulse-width. Secondly, individual quanta are expected to have well-defined trajectories. Therefore, interference should persist even in special settings, in which only one slit is accessible at a time, provided quanta have alternative access to more than one opening. At least for the situations that involve photons, both of these predictions are in agreement with the experimental record.

Several preliminary remarks are in order, before we look at the data. When a classical wave hits an obstacle with two openings, it will come out on the other side in the form of two waves, displaying interference in their area of overlap. In this sense, the original wave can be described as interfering with itself. Optical interference confirms the wave properties of light. Given this, should we expect the quanta of light to interfere with themselves? If quantized waves were similar to classical waves, then we should always expect self-interference. Yet, if they were constantly produced as discrete oscillations by propagating localized sources, self-interference should be impossible. Every localized source could only go through one slit, and the waves (defined here as space-time perturbations) could not be reflected by material obstacles. Both of these possibilities can be resolved empirically, by looking for evidence of interference at extremely low energy levels.

It is well-known that a single quantum cannot produce fringes. It can only produce a detection click. In order to observe interference, large numbers of coherent photons must be detected. Still, it is possible to determine if interaction prior to detection plays any role in the final outcome. For this, we must ensure that photons do not overlap in transit and see if they display first-order interference at arbitrary intervals between single pulses. Firstly, single-photon pulses must have finite duration. Otherwise, they will always overlap, no matter the time difference between any two detections. So, they must be chopped, or emitted in discrete pulses. Secondly, the coherence time of the source must exceed the interval between any two pulses at emission; otherwise the main pre-condition for interference will not be met. If these technical features are guaranteed, and interference persists at any rate of emission, then we can be confident about the reality of self-interference.

There are numerous experimental proofs of interference at very low rates of emission. However, the two conditions mentioned above (especially pulse-width) were not explicitly enforced in most cases. And in the few cases, when they appeared to be met, interference vanished. Usually, insufficient coherence at the source is suspected in such cases. In our opinion, the evidence does not justify such an interpretation. For example, Dontsov and Baz [10] suggested that discharge tubes, used as sources of photons, cannot produce coherent light at low levels of excitation. They demonstrated this by proving that interference vanished, when their source was weak. Furthermore, interference fringes reappeared, when they increased the output of the sources by two orders of magnitude. Nevertheless, when they placed neutral density filters behind the source, diminishing the rate of detection to the same low levels, visibility dropped again. It is remarkable that Dontsov and Baz were able to recover the fringes by moving the filters beyond the interference volume, in front of the detectors. Thus, interference visibility was independent from the technical state of the detectors, as well as from that of the source. The main factor was the number of photons passing through the interference volume per unit of time.

In a modern demonstration, Ribeiro and collaborators discarded the idler beam from a source of spontaneous parametric down conversion (SPDC), and performed a double-slit experiment with the signal beam [11]. They used a special set-up to achieve high rates of emission and controlled the pump (input) beams with neutral density filters. Narrow-band interference filters were used to screen for monochromatic detections only. The result was a very clear demonstration of interference at high emission rates, as well as of its gradual disappearance at lower rates. Unfortunately, Ribeiro *et al.* concluded that their source cannot produce coherent photons at low rates, without testing for alternative explanations. They could have placed a neutral density filter behind the source, just like Dontsov and Baz, in order to see if fringes can be produced by coherent photons at the same low emission rates. So, we have to look at other experiments for a proper interpretation. In one experiment, Kim *et al.* controlled the exact interval between independent signal photons emitted in pairs [12]. As the time-delay between photons was increased, first-order interference gradually vanished. This shows that the interval between the quanta was more important than the state of the source for the final outcome. Though, a possible objection might be that spontaneous sources cannot ensure phase-coherence, which could be especially important at large intervals between pulses. Still, there is another experiment, by Kim and Grice [13], in which sub-wavelength adjustments of time-delay were achieved. In these conditions, maximum visibility was made possible by ensuring phase coherence between interacting photons. Still, interference did not persist after a threshold interval between photons. This evidence is sufficient for us to conclude that self-interference did not happen in a context, in which its preconditions were met. Whatever the nature of matter waves, they do not seem to produce quantum interference via self-interaction.

It is also remarkable that interference happened even when quanta did not overlap in space, provided they were within the threshold boundaries. This is evidence of remote interaction between them, consistent with our hypothesis of space-time fluctuations from localized quantum sources. Furthermore, the interval between independent detections was larger for Dontsov and Baz [10] than for Ribeiro *et al.* [11], as expected. Longer pulses are physically closer to each other at fixed rates of emission. This element is

well supported by the experiment of Santori *et al.* [4]. They used quantum-dots as deterministic sources of photons to investigate interference between independent quanta. They clearly showed that interference visibility for comparable intervals was higher for quantum dots with longer excitation life-times, i.e. wider single-photon pulses.

A different requirement of self-interference is to have the two slits simultaneously open at any time that a photon can pass. Its function is self-evident, because the whole concept hinges on the ability of a wave to propagate from multiple secondary sources and generate interaction between its components. Thus, it must be impossible to get interference fringes with a single slit open at a time, if quantum self-interference is to prevail as a valid theoretical concept. In practice, this requirement was violated in a very convincing manner.

It is known that interference fringes do not form, when two slits are opened alternatively at a very slow rate. However, it was shown above that photons do not produce fringes at very low rates of emission even with both slits open. The only relevant settings are those, in which quanta from both paths have sufficient opportunities to interact. Such a set-up was prepared successfully by Sillitto and Wykes [14], who used an electric shutter to switch on and off the paths of a Young interferometer. Moreover, they were able to switch both openings several times before any single photon could reach the detector, even though the two paths were never open simultaneously. The estimated rate of emission was low enough to have no more than one photon at a time in the apparatus ( $10^6$  quanta/sec for a transit time of  $10^{-8}$  sec). However, the paths were controlled with a birefringent device, which means that components from the same single pulse could have accessed both trails. Such components could not have entered at the same time, and they had to be temporally distinguishable inside the interferometer. Still, they were close enough to interfere with each other. This, in our opinion, explains the high quality of the results that were obtained.

The experiment of Sillitto and Wykes was designed to test the role of uncertainty. The rate of switching exceeded the speed of the detector, making it impossible to link a click with a path. This, in terms of the Copenhagen interpretation, proves that uncertainty in our knowledge overrides physical properties. Even though photons must have been distinguishable during propagation through the interferometer, they were not observable as such. However, distinguishability should be a matter of principle, independent from the technological endowment of the observer. It should work regardless of the presence of the observer, or of its choice of detector. Moreover, the experiment also showed a strong dependency of fringe visibility on phase coherence, which was controlled by adjusting the length of one path. Accordingly, the photon had to “know” the technological limits of the observer as well as the exact length of both paths. Note that the photons also had to interfere with themselves, without being physically able to access both paths! In our opinion, the hypothesis of second-order quantum interaction via transient space-time fluctuations is much more credible for this experiment. Moreover, there are other tests which confirm our assessment of self-interference.

A very instructive version of the double-slit experiment was performed by Basano and Ottonello [5]. They used two independent lasers, well isolated from each other, in order to exclude any interaction between them, or between the photons, prior to interference. The beams of each source were prepared such as to access only one of two slits. Thus, every single photon had to pass through one slit at the most (or

be extinguished at the screen). High visibility interference was achieved. Again, the experiment allows for speculations that our lack of knowledge somehow “washed out” the physical parameters of individual photons. Other experiments, nevertheless, are adding up to close this loophole. Santori *et al.* [4] have demonstrated photon bunching with independent deterministic sources. In their set-up, single photons were emitted on demand and their paths through the interferometer were well-known. The photons had to have clear physical properties, even though their identity was lost at the detectors.

A possible objection to these last examples is that they refer to experiments with multiple sources. Yet, there is another experiment, in which Fonseca *et al.* demonstrated the so-called non-local double-slit interference [15]. Entangled co-propagating photons from a single source were shown to produce fringes at independent detectors, without ever crossing paths. As a result, path knowledge was complete for all the photons that produced fringes in the coincidence count regime. Again, objections could be raised that entanglement is a special case that does not apply to this context. However, it is precisely our claim that all types of Young interference are multiple-photon phenomena expressing the same underlying physical process. Entangled quanta must also behave as localized entities, interacting through their associated waves. If our conclusion is wrong, then the experiment of Fonseca *et al.* should work with counter-propagating photons as well (see below). We hope that such an experiment will be performed in the near future. In any event, the quoted evidence cannot be reconciled with the assumption of self-interference. This is a strong argument in favor of  $p$ -adic valued probability analysis of quantum interference, which predicted these kinds of findings. Theory and experiment appear to converge on the conclusion that double-slit distributions are not reducible to single-entity phenomena.

## DISCUSSION

The wave properties of optical rays can be explained with great accuracy by invoking Huygens’ Principle. According to the latter, every point on a wave-front can be treated as a source of secondary waves. On the downside, this approach was known to have difficulties explaining the well-defined directions of light waves, as well as the quantized nature of electromagnetic energy. However, if quanta are treated as localized propagating particles, generating space-time perturbations, then both of these problems can be circumvented. Particles become the unobservable causal substratum for the electromagnetic waves. (The implication is that electromagnetic interactions could be analyzed in terms of dynamic effects on the curvature of space-time). This could also explain the correlations between detection events in a double-slit experiment, as suggested by Khrennikov’s interpretation of negative probabilities. Quanta could generate discrete detections, while still being under the influence of each other’s waves, without violating classical causality. This hypothesis is quite easy to verify.

Pure waves can generate fringes only within their interference volume. Guided quanta, on the other hand, must maintain their momentum even beyond such regions. Is there any reason to believe that fringes are observable beyond interference volumes? In our assessment, this phenomenon is fully demonstrated by the so-called position correlations, displayed by pairs of co-propagating photons during their coincident detection at sepa-

rate detectors. As demonstrated for the first time by Hanbury-Brown and Twiss (HBT), such correlations are observable at arbitrary distances from the area of co-propagation, and show up even at unequal distances between the detectors and the beam-splitter [16]. The experiment of HBT is widely believed to be a consequence of classical intensity correlations. However, this hypothesis has very specific experimental consequences, which did not materialize in a recent attempt to verify them [17].

Position correlations are an essential element of quantum imaging. The latter phenomenon was also believed to depend on entanglement at the source. However, several recent experiments have demonstrated quantum imaging with chaotic sources of light [18, 19, 20, 21]. It should be noted that experiments with chaotic sources of light were only able to produce quantum effects with low visibility. Still, they raise an important question. Is entanglement essential for quantum effects, or is it just a superior technique for generating reproducible states? We would like to propose an experimental solution to this problem. According to our interpretation, fringe build-up requires (roughly) co-propagating photons. To the best of our knowledge, quantum imaging, quantum erasure, and other interference-dependent phenomena have only been demonstrated with set-ups that began with co-propagating photons. Accordingly, counter-propagating entangled photons should not be able to reproduce these effects with the same visibility. At the same time, even independent chaotic beams should be able to produce quantum effects in co-propagating arrangements. Moreover, we anticipate the possibility of achieving high-visibility quantum imaging with chaotic sources of light, well above the current 30 percent ceiling. These proposals can be fulfilled with technology that is already available in many labs. Given the high interest in this topic, we hope that such experiments will be attempted in the nearest future.

The theoretical and experimental developments discussed in this paper are opening a new perspective on the nature of quantum interference. On the one hand, we have better means to interpret the peculiarities of quantum statistics. On the other hand, we have the opportunity to test these implications with unprecedented accuracy. It is still too soon to claim that a final answer on this issue is available. However, we are persuaded that negative probabilities do not reflect any kind of anomaly in Nature. On the contrary, it is our understanding of Nature that has to improve until the mystery is solved.

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